

A.S. Rinat and M. F. Taragin

Weizmann Institute of Science, Department of Particle Physics, Rehovot 76100, Israel

(November 30, 2001)

We exploit a relationship between the Structure Functions of nucleons, the physical deuteron and of a deuteron, composed of point-nucleons to compute angular distributions of inclusive cross sections of 4.05 GeV electrons. We report general agreement with data and interpret the remaining discrepancies. We discuss the potential of the data for information on neutron structure functions $F_k^n(x, Q^2)$ and the static form factor $G_M^n(Q^2)$. 25.20.Fj,13.60.Hb

I. INTRODUCTION.

In the early days of inclusive scattering experiments the deuteron (D) as a target was only second in importance to the proton [1,2]. The obvious reason was and is the quest of information on the neutron, in particular of its static form factors $G_{E,M}^n$ and the structure functions F_k^n . Later experiments for ranges of fixed Q^2 , and similar ones for ^3He , ^4He , related mostly to the issue of scaling [3].

Modern inclusive scattering experiments [4,5] have used targets with $A \geq 4$, and only recently have data on double-differential cross sections on the D for a beam energy $E = 4.045$ GeV become available [6,7]. Those are the first of its kind, and to our knowledge no calculation has thus far been made. The incentive for such a calculation is two-fold. First, one can compute the nuclear input with great precision, making for a stringent comparison with data. Second, we shall emphasize the unusual spatial extension (or the nucleon momentum distribution) of the deuteron, which expresses itself in non-standard features in cross sections. In turn those features can be exploited to extract G_M^n [8].

II. COMPUTATION THE INCLUSIVE CROSS SECTIONS ON THE D .

We start with the expression for inclusive cross sections per nucleon of unpolarized high-energy electrons from randomly oriented arbitrary nuclear targets as function of the scattering angle θ and the energy loss of the beam ν

$$\frac{d^2\sigma^{eD}(E; \theta, \nu)/2}{d\Omega d\nu} = \frac{2}{M}\sigma_M(E; \theta, \nu) \left[\frac{xM^2}{Q^2} F_2^D(x, Q^2) + \tan^2(\theta/2) F_1^D(x, Q^2) \right], \quad (1)$$

$F_{1,2}^D(x, Q^2)$ are the Structure Functions (SF) per nucleon of the D , which may be expressed in terms of the squared 4-momentum transfer $Q^2 = \mathbf{q}^2 - \nu^2$ and the Bjorken variable x with range $0 \leq x = Q^2/2M\nu \leq 2$ (M is the nucleon mass). For given beam energy E the pairs (θ, ν) and (x, Q^2) are alternative kinematic variables.

We base calculations of inclusive cross sections on the following relation between the SF of the target F^D and of nucleons F^N [9]

$$F_k^D(x, Q^2) = \int_x^2 dz f^{PN,D}(z, Q^2) \left[F_k^p\left(\frac{x}{z}, Q^2\right) + F_k^n\left(\frac{x}{z}, Q^2\right) \right] / 2, \quad (2)$$

with $F_k^{p,n}$ the nucleon SF and $f^{PN,A}$, the SF of a nucleus composed of point-nucleons. In the expression for F_k^D for given k one ought to include coefficients, which mix different nucleon SF [10]. Their effect decrease with increasing Q^2 and we suggest that those may be neglected for the D data under investigation.

Eqs. (2) describes parton degrees of freedom of nucleons but not those, originating from other sources, for instance from virtual bosons. The latter contributions, as well as anti-screening effects decrease with increasing x , limiting the use of Eq. (2) to $x \gtrsim 0.15$ -0.20 [11], well below the smallest x reached in the data. Finally, Eq. (2) has been estimated to hold for $Q^2 \gtrsim Q_c^2 \approx 2$ -2.5 GeV² [12,13].

For use below, we mention a separation of nuclear SF, and consequently of cross sections into nucleon-elastic (NE) and nucleon-inelastic (NI) components. Those correspond to contributions which, after absorption of virtual photons, nucleons are not (NE), or are excited (NI) [13].

Eq. (2) is routinely used in the Plane Wave Impulse Approximation (PWIA) (see for instance Ref. [14]). Here we adhere to a non-perturbative version with on-shell nucleons [9] and which in the past has been applied to nuclei with $A \geq 12$ [12,15].

There are several incentives to measure and to compute inclusive scattering on light nuclei. For the time being, we recall that the nuclear specificity of SF resides in $f^{PN,A}$, which is the SF of a nucleus, composed of point-nucleons. In contrast to nuclei with $A \geq 12$, for which the nuclear part of $f^{PN,A}$ can only be computed approximately, for light nuclei such a calculation can be performed with great precision [16].

A first description of data on a light nucleus ${}^4\text{He}$, exploiting a relativistic version [17] of the Gersch-Rodriguez-Smith (GRS) theory for SF [18] has been completed [19]. Below we shall present the D case. For it, there exists only the single- N density matrix $\mathcal{A}(\mathbf{r}, \mathbf{r}')$, which in cylindrical coordinates $(\mathbf{r} = \mathbf{b}, z)$ is diagonal in \mathbf{b} . With $s = z - z'$ it reads

$$\begin{aligned}\mathcal{A}(b, z; s) &= \frac{1}{3} \sum_{M_D} \sum_{\sigma_1, \sigma_2} \langle \Phi_{M_D}(\mathbf{r} - s\hat{\mathbf{z}}_q; \sigma_1, \sigma_2) \Phi_{M_D}(\mathbf{r}; \sigma_1, \sigma_2) \rangle \\ &= \frac{1}{4\pi r r'} [u(r)u(r') + w(r)w(r')] P_2(t)\end{aligned}\quad (3)$$

Above one sums over the direction of the spin of the unpolarized D and integrates over nuclear spins (see for instance Ref. [20]).

The functions u, w in Eq. (3) are the standard radial $L=0,2$ components of the D ground state [21]. In Eq. (3) $r = \sqrt{b^2 + z^2}$, $r' = \sqrt{b^2 + (z-s)^2}$ while $P_2(t) = (3t^2 - 1)/2$, with $t = (s^2 - r^2 - r'^2)/2rr'$. The latter corresponds to the choice of the z -axis along the direction of the 3-momentum transfer \mathbf{q} .

First we choose as kinematic variables the 3-momentum transfer $|\mathbf{q}|$ and a scaling variable y , which replaces the energy loss ν . In terms of those, we decompose the relativistic reduced structure function as $\phi(q, y) = \phi_0(y) + \phi^{FSI}(q, y)$, which are the asymptotic limit and the q -dependent Final State Interactions (FSI) which perturbs the former [17]. Both employ the above one-nucleon density \mathcal{A} [12]

$$\phi_0(q) = 2\pi \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{iys} \int_0^{\infty} db b \int_{-\infty}^{\infty} dz \mathcal{A}(bz; s) \quad (4a)$$

$$\frac{M}{|\mathbf{q}|} \phi^{FSI}(q, y) = 2\pi \int_{-\infty}^{\infty} \frac{ds}{2\pi} \int_0^{\infty} db b \int_{-\infty}^{\infty} dz \mathcal{A}(bz; s) [\tilde{\Gamma}_q(bz; s) - 1] \quad (4b)$$

The FSI term for the D contains the off-shell pn scattering amplitude. Eq. (4b) above uses its eikonal approximation, which in coordinate representation is proportional to the off-shell profile $\tilde{\Gamma}_q(bz, s)$, in turn approximately related to its on-shell analog [12,22]. The latter may in a standard way be expressed in terms of elastic scattering observables

$$\tilde{\Gamma}_q(bz, s) \approx \left[1 - s \frac{\partial}{\partial s} \right] \theta(z) \theta(s - z) \Gamma_q^{(1)}(b) \quad (5a)$$

$$\Gamma_q^{(1)}(b) \approx \frac{1}{2} \sigma_q^{tot} [1 - i\tau_q] \frac{Q_0^2(q)}{4\pi} e^{-b^2 Q_0^2/4} \quad (5b)$$

Substitution into Eq. (4b) gives for the FSI part

$$\frac{M}{|\mathbf{q}|} \phi^{FSI}(q, y) \approx 2\pi \int_{-\infty}^{\infty} \frac{ds}{2\pi} \int_0^{\infty} db b \Gamma_q(b) \left[\int_0^s dz \mathcal{A}_1(bz; s) - s \mathcal{A}_1(bs; s) \right] \quad (6)$$

It is through Eq. (5a) that $\phi^{FSI}(q, y)$ acquires model-dependence.

In previous applications for $A \geq 4$ [12,15], y has been taken to be a relativistic version of the West-GRS scaling variable for $A \rightarrow \infty$ [23]

$$y_G = \frac{M\nu}{|\mathbf{q}|} \left[1 - \frac{\Delta}{M} - x \right], \quad (7)$$

with Δ some average separation energy. The above expression disregards for $A \geq 4$ the energy of the recoiling spectator, but for the D this is not accurate enough. Its inclusion leads to the replacement [17]

$$y_G \rightarrow y_G^D = M \frac{|\mathbf{q}|}{\nu} \left[\sqrt{1 + 2 \frac{\nu}{|\mathbf{q}|} \frac{y_G}{M}} - 1 \right] \quad (8)$$

In the end, one converts the structure function $\phi^D(q, y)$ into a dimensionless equivalent in terms of x, Q^2 [12,15] as required in Eq. (2)

$$f^{PN,D}(x, Q^2) = \left| \frac{\partial y_G^D}{\partial x} \right| \phi^D \left(q(x, Q^2), y_G^D(x, Q^2) \right) \quad (9)$$

In order to obtain F_k^D , the above has to be folded into $F_k^{p,n}$ (cf. Eq. (2)). Regarding the latter, there are data on F_2^p [24] and less accurate older ones for F_1^p [25]. Both do not reach the elastic region $x \lesssim 1$ but parametrizations cover the entire x -range. With no direct information on the neutron SF F_k^n , one usually assumes the 'primitive' choice $F_k^n = 2F_k^D - F_k^p$ [25,26] which corresponds to free p, n in the D .

In Fig. 1 we display total D cross sections per nucleon (1) and their NE parts for inclusive scattering of $E=4.045$ GeV electrons as function of the scattering angles $\theta = 15^\circ, 23^\circ, 30^\circ, 45^\circ, 55^\circ$ and energy loss ν . Data are from Refs. [6,7].

One notices:

1) For all scattering angles there is good agreement on the elastic side of the QEP, except for $\theta = 23^\circ$ where there is a modest disagreement for the lowest ν . This is similar to the outcome for ^4He [19], but notably different from all other targets, where low- ν predictions fail [12,15]. The general agreement there may well be due to the accuracy with which one can calculate $f^{PN,A}$ for the lightest nuclei [16] in contrast to targets with $A \geq 12$.

2) The inelastic side of the QEP is usually the one which is best produced for $A \geq 12$. However, the displayed D predictions reveal discrepancies with data, in particular for the two lowest angles. Those get less outspoken for increasing θ , degenerating in faint wiggles for $\theta = 30^\circ$. For $\theta = 45^\circ, 55^\circ$ the NI part fits very well, but the observed intensity at, and just beyond the inelastic side of the QEP, somewhat exceeds the predictions.

The failure of the underlying picture to describe the above structures is not inherent to the given description, but is a consequence of the used parametrization for $F_k^N = F_k^{N(NI)}$: Those do not account for the excitation of individual nucleon resonances and instead averages over those. Their explicit inclusion requires precise information on transition form factors $G_{M,E}^{N \rightarrow \Delta}$, which over the required Q^2 -range are not all well enough known. We therefore do not elaborate on resonance excitations beyond general statements. Yet, qualitatively one understands from Eq. (2) that $f^{PN,A}$ shifts and broadens resonance peaks. Only for the D (and then to lowest order), is the above tantamount to Fermi broadening. For higher θ (higher Q^2) the QEP and the resonance peak draw closer, get blurred and are ultimately smoothed out in the background.

We have still to account for the fact that the QEP and resonance peaks stand out for the D , but not for $A \geq 12$. The reason is the extended D , which causes the normalized $f^{PN}(x, Q^2)$ to attain a much higher maximum and corresponding narrower width than for an average nucleus. ^4He occupies an intermediate position: For $Q^2 = 3.5 \text{ GeV}^2$ the peak values of $f^{PN,A}(x, Q^2)$ for $D, ^4\text{He}$, $A \geq 12$ are 6.2, 3.1, 1.4-1.6. The above qualitatively explains the possibility to detect the outstanding QEP for the D and ^4He . The data for the latter for lower Q^2 hardly extend beyond the QEP and do barely touch on the resonance wing.

The fact that the QEP for D and ^4He are well reproduced by NE predictions makes them natural candidates to study details in the latter and its potential has been realized in the past for the neutron magnetic form factor $G_M^n(Q^2)$ [1]. We have thus finished an analysis of the QEP parts of the recent D data and of the older NE3 data on ^4He [8], constrained by new information on other static EM form factors [27].

Additional information on charge-current distributions of the neutron, contained in its Structure Function resides in the inelastic side $x \lesssim 1$ of inclusive cross sections for several, not necessarily light targets. Somewhere else we shall elaborate on the role of the D in the extraction of F_2^n [28].

III. CONCLUSION.

We have computed cross sections for inclusive scattering of 4.045 GeV electrons on D , have discussed general and exceptional features and have mentioned the potential of the data to obtain information on static form factors and dynamic structure functions of the neutron. The underlying theory is precise, but not exact and one should look forward to improvements, for instance those in the use of a Bethe-Salpeter description of the D and elastic $p - n$ scattering. Calculation of static form factors have been completed [29]). An extension to inclusive scattering would have to go beyond that model, and will somehow have to incorporate Nucleon SF as for instance in Eq. (2).

IV. ACKNOWLEDGEMENTS

The authors thank John Arrington for having provided tables of the measured cross sections.

- [1] A.Lung *et al*, Phys. Rev. Lett 70, 718 (1993).
- [2] W.P. Schuetz, Phys. Rev. Lett. 38, 259 (1977).
- [3] Rock *et al*, Phys. Rev. C26(1982) 1592; I. Sick, D. Day and J.S. McCarthy, Phys. Rev. Lett. 45, 871 (1980).
- [4] D.B. Day *et al*, Phys. Rev. C 48 1849 (1993).
- [5] J. Arrington *et al*, Phys. Rev. Lett. 82, 2056 (1999).
- [6] J. Arrington, private communication.
- [7] I. Niculescu *et al* Phys. Rev. Lett. 85, 1182 (2000).
- [8] A.S. Rinat, in preparation.
- [9] S.A. Gurvitz and A.S. Rinat, TR-PR-93-77/ WIS-93/97/Oct-PH; Progress in Nuclear and Particle Physics, Vol. 34, 245 (1995).
- [10] G.B. West, Ann. of Phys. (NY) 74, 646 (1972); W.B. Atwood and G.B. West, Phys. Rev. D7, 773 (1973).
- [11] C.H. Llewellyn Smith, Phys. Lett B 128, 107 (1983); M. Ericson and A.W. Thomas, *ibid* 112.
- [12] A.S. Rinat and M.F. Taragin, Phys Rev. C 60, 044601 (1999).
- [13] A.S. Rinat and M.F. Taragin, Phys. Rev. C 62 034602 (2000).
- [14] See for instance: C. Ciofi degli Atti, E. Pace and G. Salme, Phys. Rev. C 43, 1155 (1991).
- [15] A.S. Rinat and M.F. Taragin, Nucl. Phys. A598, 349 (1996); *ibid* A620, A620, 412 (1997); Erratum *ibid* A623, 773 (1997).
- [16] H. Kamada *et al*, Phys. Rev. C 64, 044001 (2001).
- [17] S.A. Gurvitz and A.S. Rinat, nucl-th/0106032, Phys. Rev. C 65, to be published.
- [18] H.A. Gersch, L.J. Rodriguez and Phil N. Smith, Phys. Rev. A5, 1547 (1973).
- [19] M. Viviani, A. Kievsky and A.S. Rinat, nucl-th 0111049, submitted to Phys. Rev. C.
- [20] A. Bianconi, S. Jeschonnek, N.N. Nikolaev and B.G. Zakharov, Phys. Lett. B 343, 13 (1995).
- [21] R. Machleidt, K. Hohlinde and C. Elster, Phys. Rep. 149, 1 (1987).
- [22] A.S. Rinat and M.F. Taragin, Nucl. Phys. A 623, 519 (1997).
- [23] S.A. Gurvitz, Phys. Rev. C 42, 2653 (1990).
- [24] P. Amadrauz *et al*, Phys. Lett B295, 159 (1992); M. Arneodo *et al*, *ibid* B364, 107 (1995).
- [25] A. Bodek and J. Ritchie, Phys. Rev. D 23, 1070 (1981).
- [26] M. Arneodo *et al*, Phys. Rev. D 50, R1 (1994).
- [27] M. Jones *et al*, Phys. Rev. Lett. 84, 1398 (2000); Third Workshop on 'Perspective in Hadronic Physics' Trieste 2001, IT; to be published.
- [28] A.S. Rinat and M.F. Taragin, in preparation.
- [29] C. Ciofi degli Atti, D. Faralli, A. Yu. Umnikov and L.P. Kaptari, Phys. Rev. C 60, 034003 (1999).

Figure captions

Fig. 1 Cross sections for inclusive scattering of 4.045 GeV electrons from D for $\theta = 15^\circ, 23^\circ, 30^\circ, 45^\circ, 55^\circ$ as function of the beam energy loss ν . Data are from Ref. [6].